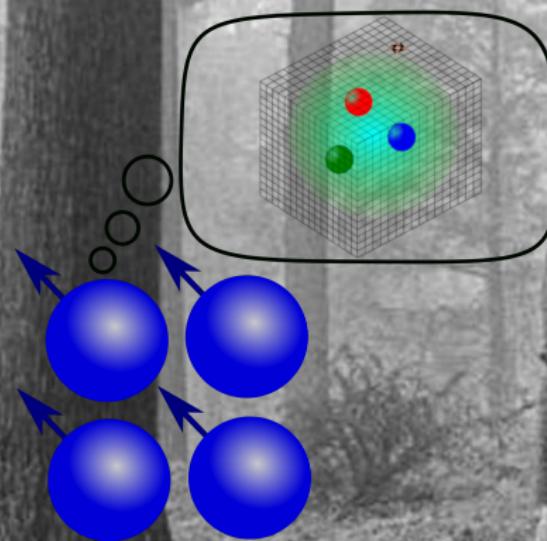


# Gluon Field Digitization for Quantum Computers

Hank Lamm

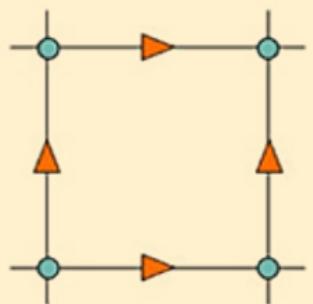
New Perspectives 2020 (2.0)



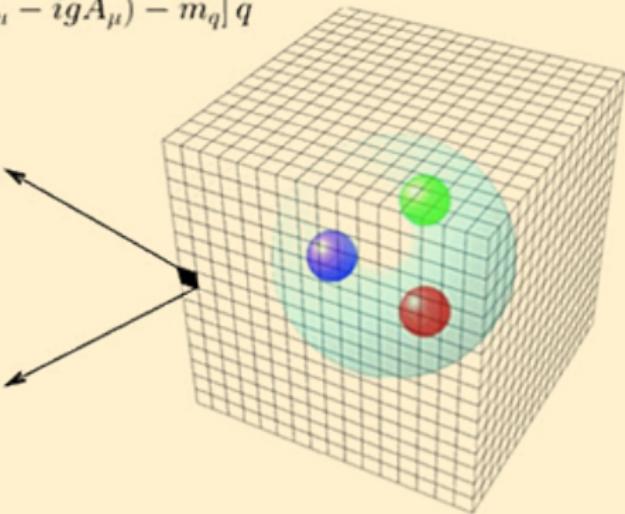
# Euclidean Lattice Field Theory is wildly successful

## QCD Lagrangian

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \sum_{q=u,d,s,c,b,t} \bar{q} [i\gamma^\mu(\partial_\mu - igA_\mu) - m_q] q$$



● quark      ▲ gluon



LFT can compute **most**  $\langle \psi_i | \prod_n \mathcal{O}_n(\tau_n) | \psi_i \rangle$

Early on, there were competing methods

Real- $t$  QFT have sign problems<sup>[1]</sup>

[1]

Feynman, R. P. In: *Int.J.Theor.Phys.* 21 (1982).

**So ahead of the curve, the curve becomes a sphere**

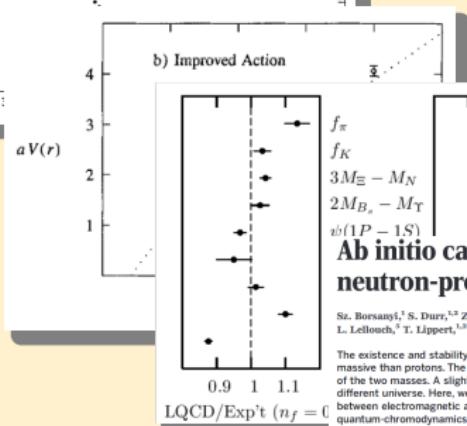
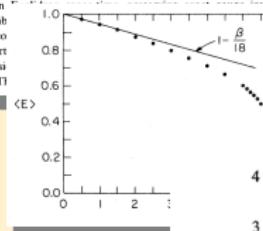
Confinement of quarks\*

Kenneth G. Wilson

Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850  
(Received 12 June 1974)

*Studies, Cornell University,*

A mechanism for total confinement of quarks, similar to that of Schwinger, is defined which requires the existence of Abelian or non-Abelian gauge fields. It is shown how to quantize a gauge field theory on a discrete lattice in fields as angular variables a computable strong-coupling expansion. There is no unfortunatly strong-coupling expansion joining path spaces. The



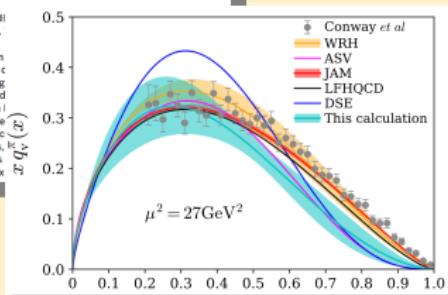
← If we're lucky, we are here<sup>†</sup>

<sup>†</sup>...except I'm no Wilson.

# Ab initio calculation of the neutron-proton mass difference

Sz. Borsanyi,<sup>1</sup> S. Durr,<sup>1,2</sup> Z. Fodor,<sup>1,2,3,\*</sup> C. H. L. Lellouch,<sup>5</sup> T. Lippert,<sup>1,3</sup> A. Portelli,<sup>5,6</sup> K.

The existence and stability of atoms rely on massive than protons. The measured mass  $c$  of the two masses. A slightly smaller or larger different universe. Here, we show that this difference between electromagnetic and mass isospin quantum-chromodynamics and quantum-electron degenerate Wilson fermion flavors and  $c$ , with an accuracy of 300 kilo-electron volts, deviations. We also determine the splittings exceeding in some cases the precision of ex-



# Lattice QCD: A forty-five year case study

# How do I digitize gluons, all things considered?

Lots of choices for bosons:

- Loop-String-Hadrons<sup>[2]</sup> – Tensor Networks<sup>[3]</sup> —Quantum Link Models<sup>[4]</sup> – **Discrete Subgroups**<sup>[5]</sup> → Matrices with floats<sup>[6]</sup>

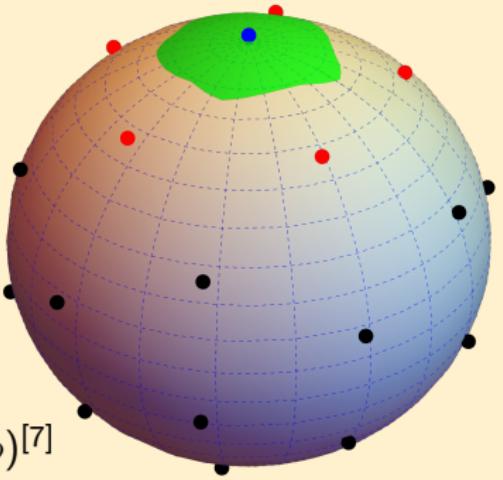
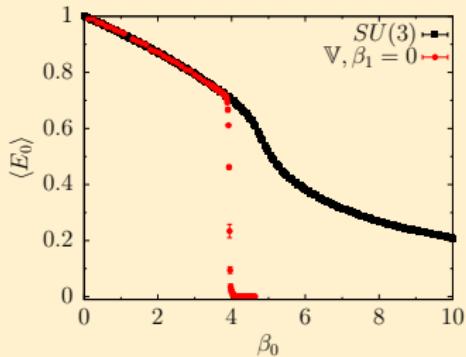
What qualities make a GOOD scheme?

- What is the qubit cost per degree of freedom?
- What is the circuit depth per trotterization step?
- What is the rate of approach to the physical point?
- How easily can they be analyzed?
- **Can the scheme be simulated classically?**

- 
- [2] Raychowdhury, I. and J. R. Stryker. In: *Phys. Rev. D* 101 (2020). arXiv: 1912.06133 [hep-lat].
  - [3] Butt, N., S. Catterall, Y. Meurice, R. Sakai, and J. Unmuth-Yockey. In: *Phys. Rev. D* 101 (2020).
  - [4] Luo, D. et al. In: (Dec. 2019). arXiv: 1912.11488 [quant-ph].
  - [5] Alexandru, A. et al. In: *Phys. Rev. D* 100 (2019). arXiv: 1906.11213 [hep-lat].
  - [6] Hackett, D. C. et al. In: *Phys. Rev. A* 99 (2019). arXiv: 1811.03629 [quant-ph].

# Discrete subgroups allow plug-and-play framework<sup>[8]</sup>

- Replace  $G \rightarrow H$  in  $e^{-S}, e^{-i\mathcal{H}}$
- $H$  has  $\Delta S > 0$  so 1st PT at  $\beta_f < \infty$ ...



- $\mathcal{L} \approx -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^\dagger(D^\mu\phi) - V(\phi)$ <sup>[7]</sup>
- UV difference scales  $\propto \left(\frac{\phi}{\Lambda}\right)^n$
- If  $\beta_f > \beta_s \implies$  rough approx. in EFT sense

[7]

Fradkin, E. H. and S. H. Shenker. In: *Phys. Rev. D* 19 (1979).

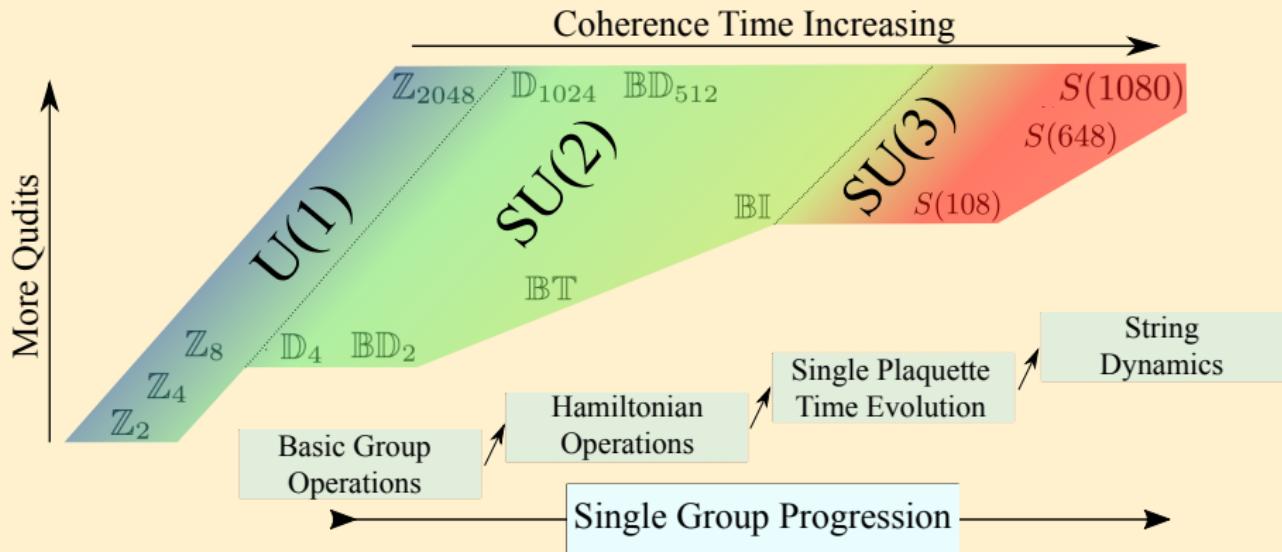
[8]

Bhanot, G. and C. Rebbi. In: *Phys. Rev. D* 24 (1981), Bhanot, G. In: *Phys. Lett.* 108B (1982), Hackett, D. C. et al. In: *Phys. Rev.* A99 (2019). arXiv: 1811.03629 [quant-ph].

# A Ladder from Discrete Subgroups

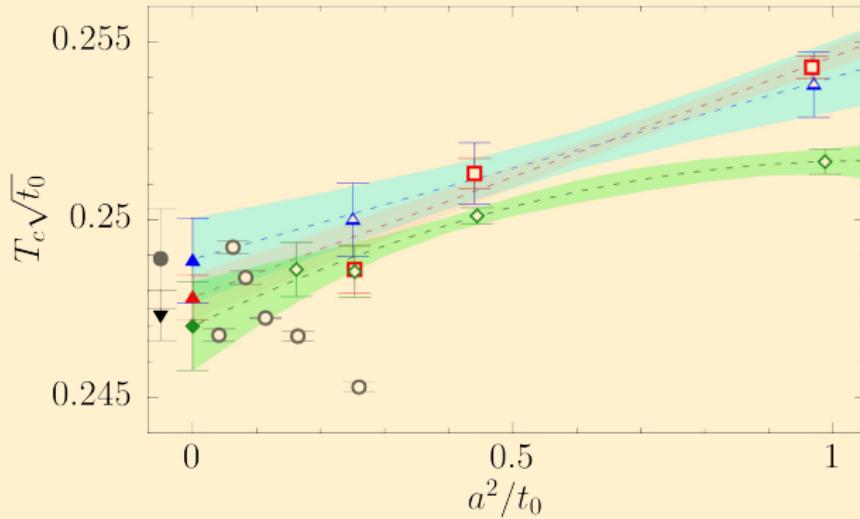
What do we know from Wilson Action?

- $U(1) \rightarrow \mathbb{Z}_N, N > 4$
- $SU(2) \rightarrow \mathbb{BO}, \mathbb{BI}$
- $SU(3) \rightarrow \mathbb{V}$  has  $\beta_f = 3.935(5) < \beta_s \approx 6$
- One **1152** qubit  $SU(3)$  link vs  $\sim 4^3$  lattice of **11** qubits for  $\mathbb{V}$  link



$T_c\sqrt{t_0}$  suggests  $a \approx 0.07$  fm  $\approx 2$  GeV $^{-1}$  possible<sup>[10]</sup>

$S = \sum \frac{\beta_0}{3} \operatorname{Re} \operatorname{Tr} U + \beta_1 f(U)$  with  $f(U) = \{\operatorname{Tr}^2 U + \operatorname{Tr} U^2, |\operatorname{Tr} U|^2\}$   
Compare to SU(3)<sup>[9]</sup>



**On-going** work to extract quenched spectroscopy

[9]

Francis, A., O. Kaczmarek, M. Laine, T. Neuhaus, and H. Ohno. In: *Phys. Rev. D* 91 (2015). arXiv: 1503.05652 [hep-lat].

[10]

Alexandru, A. et al. In: *Phys. Rev. D* 100 (2019). arXiv: 1906.11213 [hep-lat].

# Systematics from Group Space Decimation<sup>[11]</sup>

Decimate via  $U = u \cdot \epsilon$  in analogy to Wilsonian renormalization:

$$Z = \int_G D\mathbf{U} e^{-S[\mathbf{U}]} = \sum_{u \in H} \int_{\Omega} D\epsilon e^{-S[u, \epsilon]} = \sum_{u \in H} e^{-S[u]}, \quad (1)$$

Match moments with cumulants in terms of  $\chi_r$  with  $V_r \propto \langle \chi_r(\epsilon) \rangle$

$$\begin{aligned} S[u] = & \sum_p (\beta_{\{1\}} + \beta_{\{1,1\}}) \frac{1}{3} \operatorname{Re} \chi_{\{1\}} + (\beta_{\{0\}} + \beta_{\{1,1,1\}}) \\ & + (\beta_{\{2\}} + \beta_{\{1,1,-1\}}) \frac{1}{6} \operatorname{Re} \chi_{\{2\}} + (\beta_{\{1,-1\}} + \beta_{\{2,1\}}) \frac{1}{8} \chi_{\{1,-1\}} \\ & + \frac{\beta_{\{3\}}}{10} \operatorname{Re} \chi_{\{3\}} + \frac{\beta_{\{2,-1\}}}{15} \operatorname{Re} \chi_{\{2,-1\}}. \end{aligned} \quad (2)$$

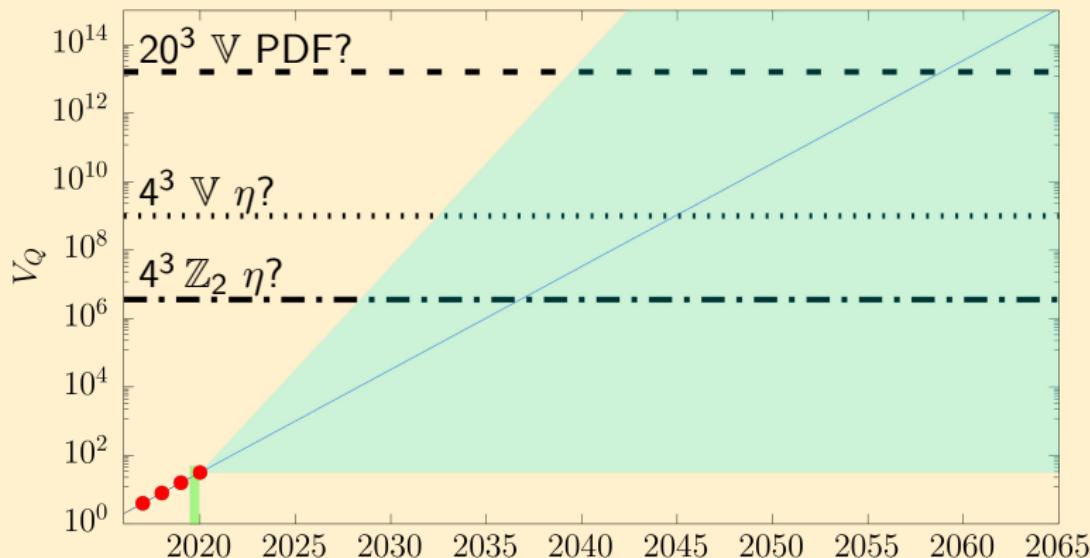
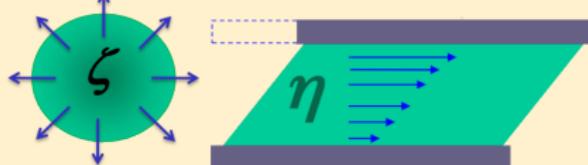
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[11]

Flyvbjerg, H. In: *Nucl. Phys.* B243 (1984), Ji, Y., H. Lamm, and S. Zhu. In: (May 2020). arXiv: 2005.14221 [hep-lat].

# What will it take for practical quantum advantage?

- Find **Bulk properties** with **fast continuum limit**  $\implies$  smaller  $V$
- Analogs to specific heat are **viscosity/conductivity**



# We are at the start of an exciting era!

Many things to do!

- Digitizing SU(3)
  - Spectroscopy for  $\mathbb{V}$
  - $\mathbb{V}$  circuits
- Error Analysis
  - e.g. Finite volume, decimation errors, fidelity to obtain **realistic** resource estimates
- Investigate desirable properties
  - **Viscosity?**

